

# Charge-detector-induced backaction and full counting statistics of transport through a double quantum dot

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Full counting statistics (FCS) of electron transport is of fundamental importance for a deeper understanding of the underlying physical processes in quantum transport in nanoscale devices. Here we investigate the backaction of a charge detector in the form of a quantum point contact (QPC) on the FCS of a biased double quantum dot (DQD). We show that this inevitable QPC-induced backaction can have profound effects on the FCS under certain conditions, namely enhancing current flow through the DQD, and changing the shot noise from being sub-Poissonian to super-Poissonian.

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## I. INTRODUCTION

Current fluctuations in nanoscale systems provide key insights into the nature of charge transfer beyond what is obtainable from a conductance measurement alone (see, e.g., Refs. 1 and 2 for recent reviews). An in-depth understanding, however, may require us to go beyond the first-order and even the second-order current correlation functions (corresponding to the average current and shot noise respectively) to study the full counting statistics (FCS)<sup>3,4</sup> which yields all zero-frequency correlation functions at once. Real-time detection of the tunneling of individual electrons, an important step towards experimental measurement of the FCS, has recently been achieved in various quantum-dot (QD) systems.<sup>5-7</sup> In particular, since its measurement in a single QD for the first time,<sup>8</sup> FCS has become an important experimental tool to examine interaction and coherence effects in nanoscale systems under out-of-equilibrium conditions.<sup>9-13</sup> More recently, FCS was applied to characterize correlations in both classical and quantum systems.<sup>14</sup>

However, a pronounced effect known as backaction on the FCS of electron transport<sup>2,15</sup> is inevitably introduced during measurements made by even the most noninvasive detectors such as a quantum point contact (QPC).<sup>16</sup> Very recently, such backaction has been investigated experimentally in a single QD.<sup>16,17</sup> In contrast to a single QD, a double quantum dot (DQD)<sup>18</sup> involves coherent coupling between two different dots, and therefore can be used to demonstrate prominent coherent effects. The FCS for DQDs has been studied theoretically<sup>19</sup> and experimentally only for noise properties.<sup>20,21</sup> In addition, for a zero-bias DQD, the effects of charge-detector-induced backaction was studied theoretically<sup>22</sup> to explain experimental observations on inelastic electron tunneling.<sup>23</sup> However, to the best of our knowledge, the impacts of charge-detector-induced backaction on the FCS in these QD systems have not yet been studied.

In this paper, we investigate the FCS of electron transport through a biased DQD under measurement by a charge detector. We demonstrate that this inevitable backaction can indeed have profound effects on the FCS under certain conditions. In particular, it enhances the current flow through the DQD, and can change the nature of the shot noise from being sub-Poissonian to super-Poissonian. These QPC-backaction-induced effects are expected to be experimentally observable with currently existing technologies. Apart from a deeper understanding of experimental observations, this study may also shed light on how to control quantum transport processes in these QD systems. Also, we show that when the measuring device is absent (i.e., when the charge detector is decoupled from the DQD), we recover the known results for both current<sup>24</sup> and shot noise.<sup>25</sup>

This paper is organized as follows. In Sec. II, the model is explained and the formulas for calculating the FCS are derived. In Sec. III, the backaction by the charge detector is analyzed. Summary is given in Sec. IV.

## II. FULL COUNTING STATISTICS OF A DOUBLE QUANTUM DOT

We focus on a setup consisting of a lateral DQD, which is coupled to the source and drain electrodes, and measured by a nearby QPC [see Fig. 1(a)]. The lateral DQD is formed by properly tuning the voltages applied to the corresponding gates. Here we consider a Coulomb-blockade regime with strong intradot and interdot Coulomb interactions, so that only one electron is allowed in the DQD system. The states of the DQD are represented by the occupation states  $|1\rangle$  and  $|2\rangle$ , denoting one electron in the left and the right dots, respectively [see Fig. 1(b)].

The total Hamiltonian of the whole system can be writ-

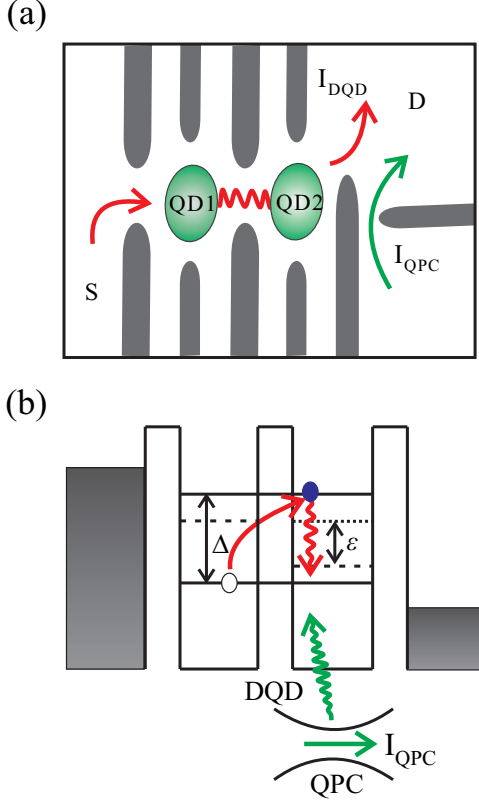


FIG. 1: (Color online) (a) Schematic diagram of a DQD coupled to two electrodes (S and D) via tunneling barriers. A QPC used for measuring the DQD electron states also yields backaction on the DQD. (b) Electronic transition between two eigenstates  $|g\rangle$  and  $|e\rangle$  of the DQD (with an energy difference  $\Delta$ ) can be induced by the charge detector QPC. The energy detuning  $\varepsilon$  between the two single-dot levels (dashed lines) can be varied by tuning the gate voltages.

ten as

$$H = H_{\text{DQD}} + H_{\text{leads}} + H_{\text{QPC}} + H_{\text{T}} + H_{\text{det}}, \quad (1)$$

where (we set  $\hbar = 1$ )

$$H_{\text{DQD}} = \frac{\varepsilon}{2} \sigma_z + \Omega \sigma_x, \quad (2)$$

$$H_{\text{leads}} = \sum_s (\omega_{ls} c_{ls}^\dagger c_{ls} + \omega_{rs} c_{rs}^\dagger c_{rs}), \quad (3)$$

$$H_{\text{QPC}} = \sum_{kq} (\omega_{Sk} c_{Sk}^\dagger c_{Sk} + \omega_{Dq} c_{Dq}^\dagger c_{Dq}) \quad (4)$$

are, respectively, the free Hamiltonians of the DQD, the electrodes coupled to the DQD, and the QPC without the tunneling term. In the DQD Hamiltonian,  $\varepsilon$  is the energy detuning between the two single-dot levels and  $\Omega$  is the interdot tunneling-coupling strength. Also,

we define pseudospin operators  $\sigma_z \equiv a_2^\dagger a_2 - a_1^\dagger a_1$  and  $\sigma_x \equiv a_2^\dagger a_1 + a_1^\dagger a_2$ , with  $a_1$  ( $a_2$ ) being the annihilation operator for an electron staying at the left (right) dot.  $c_{ls}$  ( $c_{rs}$ ) is the annihilation operator for electrons in the source (drain) reservoir, i.e., the left (right) electrode of the DQD, while  $c_{Sk}$  ( $c_{Dq}$ ) is the annihilation operator for electrons in the source (drain) reservoir of the QPC with momentum  $k$  ( $q$ ). The tunneling-coupling Hamiltonian between the DQD and the two electrodes is

$$H_{\text{T}} = \sum_s [(\Omega_{ls} c_{ls}^\dagger a_1 + \Omega_{rs} \Upsilon_r^\dagger c_{rs}^\dagger a_2) + \text{H.c.}], \quad (5)$$

where the counting operator  $\Upsilon_r$  ( $\Upsilon_r^\dagger$ ) decreases (increases) the number of electrons that have tunneled into the right electrode (via the barrier between the DQD and the right electrode).<sup>26</sup> They are introduced to keep track of the progress of the tunneling processes by successive electrons. Finally,

$$H_{\text{det}} = \sum_{kq} (T - \zeta \sigma_z) (c_{Sk}^\dagger c_{Dq} + c_{Dq}^\dagger c_{Sk}), \quad (6)$$

describes tunnelings in the QPC. It depends on the electron occupation of the DQD, owing to electrostatic couplings between the DQD and the QPC. We define  $T \equiv T_0 - (\zeta_2 + \zeta_1)/2$  and  $\zeta \equiv (\zeta_2 - \zeta_1)/2$ , so that the transition amplitudes of the QPC, when an extra electron staying at the left and right dots, equals  $T + \zeta$  and  $T - \zeta$ , respectively.<sup>27</sup>

To study the FCS of the transport through the DQD system, it is essential to know the probability  $P(n, t)$  of  $n$  electrons having been transported from the DQD to the right electrode during a period of time  $t$ . It is related to the cumulant generating function (CGF)  $G(\chi)$  defined by<sup>2</sup>

$$e^{-G(\chi)} = \sum_n P(n, t) e^{i\chi n}. \quad (7)$$

We will consider the time interval  $t$  much longer than the tunneling time of an electron through the DQD system, so that transient properties are insignificant. The derivative of the CGF with respect to the counting field  $\chi$  at  $\chi = 0$  yields the  $j$ -th cumulant, i.e.  $C_j = -(-i\partial_\chi)^j G(\chi)|_{\chi \rightarrow 0}$ , where  $\chi$  is a field conjugate to  $n$  (see, e.g., Ref. 2). These cumulants carry complete information on the FCS of the DQD system. For instance, the average current and the shot noise can be expressed as  $I = eC_1/t$  and  $S = 2e^2C_2/t$ . Thus, the Fano factor  $F$ , which is used to characterize the bunching and anti-bunching phenomena in the transport processes, is given by  $F = S/2eI = C_2/C_1$ . The asymmetry (or skewness) of the distribution is given by the third-order cumulant  $C_3$ .

On the other hand, the probability distribution function of the number of transported charges can also be expressed as

$$P(n, t) = \rho_{00}^n(t) + \rho_{gg}^n(t) + \rho_{ee}^n(t), \quad (8)$$

where  $\rho_{ij}^n(t)$  ( $i, j \in \{0, g, e\}$ ) denote the reduced density matrix elements of the DQD at a given number  $n$  of electrons transported from the DQD to the right electrode in time  $t$ . Here 0,  $g$ , and  $e$  denote the eigenstates  $|0\rangle$ ,  $|g\rangle$ , and  $|e\rangle$  of the DQD, which corresponds to no electron staying in the DQD, one electron in the ground state, and one electron in the excited state, respectively [see horizontal solid lines in Fig. 1(b)]. These reduced density matrix elements satisfy a master equation derived in Appendix A. Fourier transforming the reduced density matrix elements using

$$\rho_{ij}(\chi, t) = \sum_n \rho_{ij}^n(t) e^{i\chi n}, \quad (9)$$

we can convert the master equation to

$$\frac{\partial \varrho}{\partial t} = -\mathcal{M}(\chi)\varrho, \quad (10)$$

with

$$\mathcal{M}(\chi) = \begin{pmatrix} \Gamma_L & -\beta^2 \Gamma_R e^{i\chi} & -\alpha^2 \Gamma_R e^{i\chi} & 2\alpha\beta \Gamma_R e^{i\chi} & 0 \\ -\alpha^2 \Gamma_L & \beta^2 \Gamma_R + \gamma_{\text{ex}} & -\gamma_{\text{re}} & -\alpha\beta \Gamma_R - 2\eta\gamma_{\text{de}} & 0 \\ -\beta^2 \Gamma_L & -\gamma_{\text{ex}} & \alpha^2 \Gamma_R + \gamma_{\text{re}} & -\alpha\beta \Gamma_R + 2\eta\gamma_{\text{de}} & 0 \\ -\alpha\beta \Gamma_L & -\frac{1}{2}\alpha\beta \Gamma_R - \eta\gamma_{\text{ex}} & -\frac{1}{2}\alpha\beta \Gamma_R + \eta\gamma_{\text{re}} & \frac{1}{2}\Gamma_R + 2\eta^2\gamma_{\text{de}} & -\Delta \\ 0 & 0 & 0 & \Delta & \frac{1}{2}\Gamma_R + 2\eta^2\gamma_{\text{de}} - \gamma_{\text{ex}} - \gamma_{\text{re}} \end{pmatrix}, \quad (11)$$

where  $\varrho \equiv (\rho_{00}(\chi, t), \rho_{gg}(\chi, t), \rho_{ee}(\chi, t), \text{Re}[\rho_{eg}(\chi, t)], \text{Im}[\rho_{eg}(\chi, t)])^T$ . In the matrix  $\mathcal{M}(\chi)$ ,  $\alpha = \cos(\theta/2)$ ,  $\beta = \sin(\theta/2)$ , and  $\eta = \cos\theta$ , with  $\tan\theta = 2\Omega/\varepsilon$ ;  $\Delta = \sqrt{\varepsilon^2 + 4\Omega^2}$ , and  $\Gamma_{L(R)} = 2\pi g_{L(R)} \Omega_{lk(rk)}^2$  is the rate of electron tunneling through the barrier between the DQD and the left (right) electrode. Here,  $g_i$  ( $i = L$ , or  $R$ ) denotes the density of states at the left or right electrode of the DQD, which is assumed to be constant over the relevant energy range. The QPC-induced excitation rate  $\gamma_{\text{ex}}$ , relaxation rate  $\gamma_{\text{re}}$ , and dephasing rate  $\gamma_{\text{de}}$  are given by

$$\begin{aligned} \gamma_{\text{ex}} &= \lambda [\Theta(eV_{\text{QPC}} - \Delta) + \Theta(-eV_{\text{QPC}} - \Delta)], \\ \gamma_{\text{re}} &= \lambda [\Theta(eV_{\text{QPC}} + \Delta) + \Theta(-eV_{\text{QPC}} + \Delta)], \\ \gamma_{\text{de}} &= \lambda [\Theta(eV_{\text{QPC}}) + \Theta(-eV_{\text{QPC}})], \end{aligned} \quad (12)$$

where  $\lambda = 2\pi g_S g_D \zeta^2$ , with  $g_S$  ( $g_D$ ) being the density of states of the source (drain) electrode in the QPC. We also assume  $g_S$  and  $g_D$  to be constant over the relevant energies.<sup>28</sup> Finally,  $\Theta(x) = (x + |x|)/2$ .

The dynamical evolution of  $\varrho(\chi, t)$  at long time  $t$  is governed by  $\dot{\varrho} = -\Lambda_{\min}(\chi)\varrho$ , where  $\Lambda_{\min}(\chi) (\geq 0)$  is the minimal eigenvalue of  $\mathcal{M}(\chi)$ . Thus,

$$\begin{aligned} \sum_n P(n, t) e^{i\chi n} &\equiv \rho_{00}(\chi, t) + \rho_{gg}(\chi, t) + \rho_{ee}(\chi, t) \\ &= C e^{-\Lambda_{\min}(\chi)t}. \end{aligned} \quad (13)$$

Since probability normalization implies  $\sum_n P(n, t_0) = 1$ , we have  $C = 1$  and  $\Lambda_{\min} \rightarrow 0$  as  $\chi \rightarrow 0$ . Then, from Eq. (7), the generating function  $G(\chi)$  can be obtained from<sup>31</sup>  $G(\chi) = \Lambda_{\min} t$ .

With the transition rate of a single electron hopping from one reservoir of the QPC to the other, we can use

the Landauer formula to obtain the transition probability<sup>29</sup> and then have  $g_S g_D = \tilde{I}_{\text{QPC}} \hbar / 2\pi T^2 e^2 \tilde{V}_{\text{QPC}}$ , where  $\tilde{I}_{\text{QPC}}$  and  $\tilde{V}_{\text{QPC}}$  are the current and bias voltage of the QPC with densities of states  $g_S$  and  $g_D$  at the source and drain reservoirs. Following a recent experiment reported in Ref. 30, we take  $\tilde{I} = 500$  nA and  $\tilde{V}_{\text{QPC}} = 0.5$  meV. In addition, we choose  $T = 0.5$  and  $\zeta = 0.022$ , as in Ref. 23, so that the QPC conductance changes by  $\sim 1\%$  if the number of electrons in the DQD changes by one.<sup>30</sup>

### III. CHARGE-DETECTOR-INDUCED BACKACTION

In order to obtain compact analytical results for the FCS, we consider, for simplicity, the case where energy difference between two single-dot levels is zero (i.e.,  $\varepsilon = 0$ ). For instance, the charge current through the DQD is obtained as

$$I = \frac{4e\Gamma_L \Gamma_R \Omega^2}{\Xi}, \quad (14)$$

and shot noise is

$$S = \frac{8e^2 \Gamma_L \Gamma_R \Omega^2 f}{\Xi^3}, \quad (15)$$

where

$$\Xi = \Gamma_L \Gamma_R^2 + 4(2\Gamma_L + \Gamma_R)\Omega^2 - 2\Gamma_L \Gamma_R (\gamma_{\text{ex}} + \gamma_{\text{re}}), \quad (16)$$

$$\begin{aligned} f &= 16\Gamma_R^2 \Omega^4 + \Gamma_L^2 (\Gamma_R^4 - 8\Gamma_R^2 \Omega^2 + 64\Omega^4) + 4\Gamma_L^2 \Gamma_R \\ &\quad \times (\gamma_{\text{ex}} + \gamma_{\text{re}}) [-\Gamma_R^2 - 4\Omega^2 + \Gamma_R (\gamma_{\text{ex}} + \gamma_{\text{re}})]. \end{aligned} \quad (17)$$

Then, the Fano factor  $F \equiv S/2eI$  also follows straightforwardly.

From Eqs. (14)-(17), it is clear that the charge current  $I$ , the shot noise  $S$ , and hence also the Fano factor  $F$  depend on the excitation rate  $\gamma_{\text{ex}}$  and the relaxation rate  $\gamma_{\text{re}}$  induced by the QPC. These reveal the impacts of the backaction from the charge detector. More importantly, because of the nontrivial dependence of both  $\gamma_{\text{ex}}$  and  $\gamma_{\text{re}}$  on the applied voltage across the QPC, the presence of the charge-detector-induced backaction could be experimentally checked more easily. This will be explained in detailed in the following. In addition, as a comparison, we also consider the no-backaction cases in which  $\gamma_{\text{ex}}$  and  $\gamma_{\text{re}}$  in Eqs. (14) and (15) equal zero. These no-backaction results are in accord with previous results given by other approaches.<sup>24,25</sup> In our calculations, parameters like the interdot coupling strength  $\Omega$  and the tunneling rate  $\Gamma_L$  are taken from the experimental data whenever possible.<sup>20,21</sup>

The charge current obtained from the cumulant  $C_1$  of the FCS is calculated both with and without backaction and the results are presented in Fig. 2. When the backaction from the charge detector is taken into account, we observe that the current through the DQD is significantly enhanced. In particular, when  $|eV_{\text{QPC}}| \leq \Delta$ , a plateau with a constant current is observed. This plateau corresponds to a regime in which QPC-induced excitations is suppressed while there is still a constant relaxation rate contributed by the presence of the QPC, as can be interpreted from Eq. (12). Physically, a critical energy  $\Delta$  exists for the QPC-induced excitation of an electron in the DQD and is hence required to change the current.<sup>22,23</sup> Beyond the constant regime, that is,  $|eV_{\text{QPC}}| > \Delta$ , it is clearly shown that the current increases with the magnitude of the voltage applied across the QPC.

Compared with the case of  $\Gamma_R = 0.05 \text{ meV}$  [see Fig. 2(a)], if the rate of electron tunneling to the right electrode is tuned to larger values, e.g.,  $\Gamma_R = 0.15 \text{ meV}$  [Fig. 2(b)] and  $0.25 \text{ meV}$  [Fig. 2(c)], the current enhancement due to the backaction becomes more pronounced. This behavior may be tractable by directly examining Eq. (14). Physically, it can be understood since an essential precondition for current flow in the unidirectional electron transport [see Fig. 1(a)] is dominated by the interaction between the DQD and the right electrode.

For the Fano factor  $F \equiv S/2eI$ , our findings are given in Fig. 3. Without backaction, e.g., when the QPC is decoupled to the DQD, the Fano factor is always smaller than 1 (see dotted lines in Fig. 3). This means that antibunching of electrons corresponding to *sub*-Poissonian noise is expected (the physical reasons will be discussed below). If QPC-induced backaction is considered but with a condition  $|eV_{\text{QPC}}| \leq \Delta$ , we find a plateau similar to that in the current. As mentioned above, the electron transport in this regime does not involve QPC-induced excitations. Beyond the threshold (i.e.  $|eV_{\text{QPC}}| > \Delta$ ), the Fano factor increases with  $|V_{\text{QPC}}|$ . For a sufficiently large bias, we can get  $F = 1$  [see Figs. 2(b) and 2(c)],

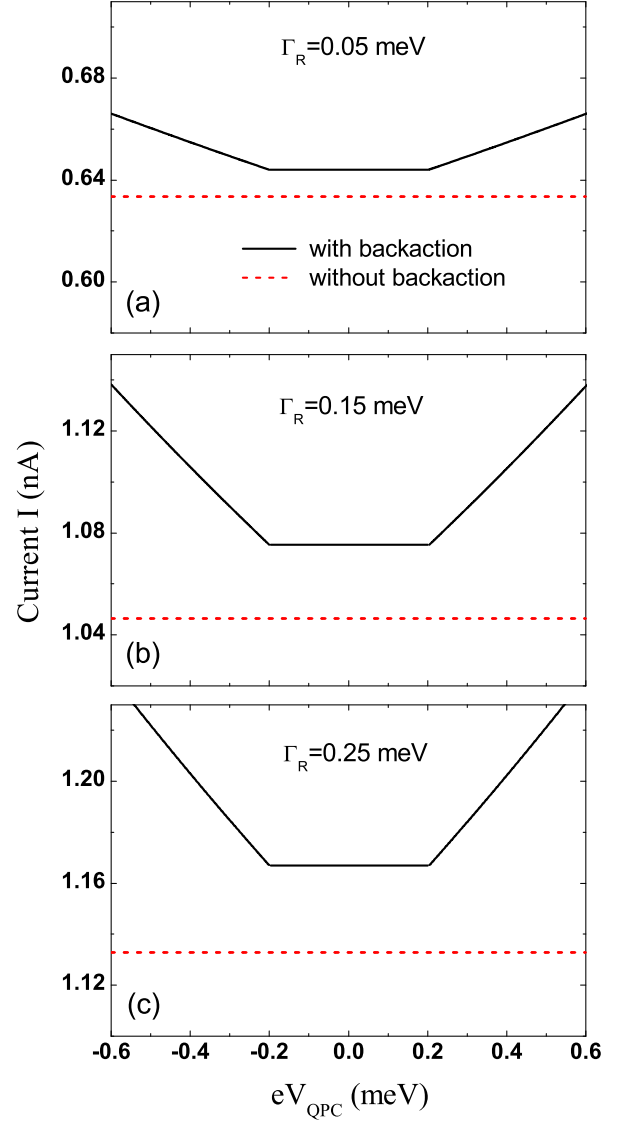


FIG. 2: (Color online). Current  $I$  versus the QPC bias energy  $eV_{\text{QPC}}$  for tunneling rate  $\Gamma_R$  equal to: (a)  $0.05 \text{ meV}$ , (b)  $0.15 \text{ meV}$ , (c)  $0.25 \text{ meV}$ . We use typical experimental parameters  $\Omega = 0.1 \text{ meV}$  and  $\Gamma_L = 0.05 \text{ meV}$  from Refs. 20 and 21.

indicating that the electron transport is uncorrelated in time and is described by *Poissonian* statistics. Beyond this second threshold,  $F > 1$ . Thus, bunching of electrons in the transport through the DQD occurs, resulting in *super*-Poissonian noise. This QPC-induced change of shot noise from being sub-Poissonian ( $F < 1$ ) to super-Poissonian ( $F > 1$ ), and vice versa, becomes more pronounced with a smaller threshold voltage when the tunneling rate  $\Gamma_R$  is increased.

Besides the charge-detector-induced backaction discussed above, one might expect that dynamical channel blockade could also be responsible for the super-Poissonian noise. In the following we rule out this pos-

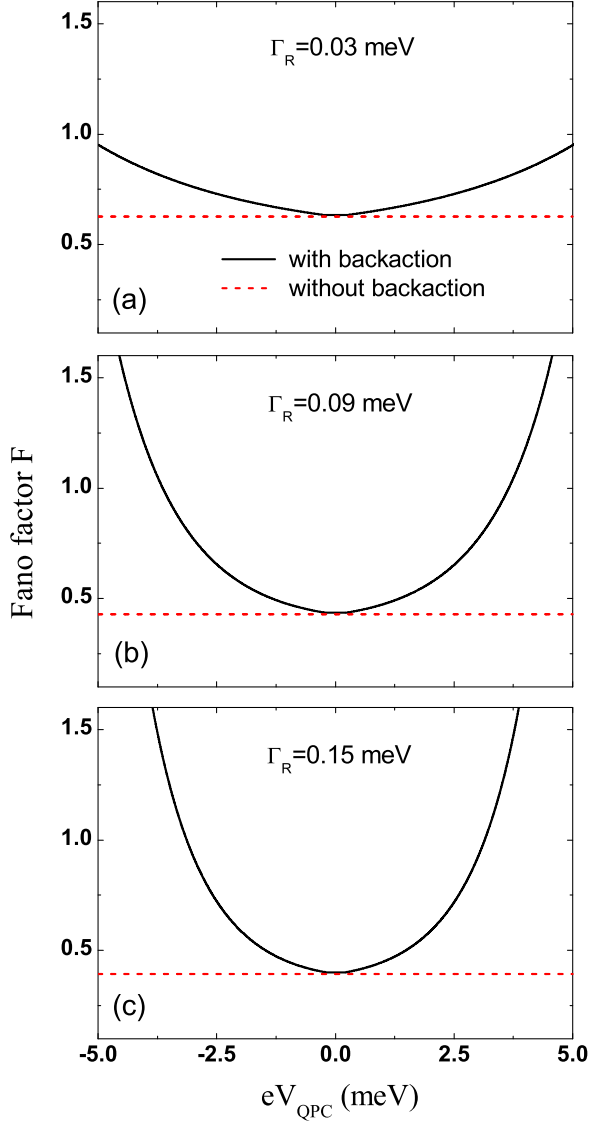


FIG. 3: (Color online). Fano factor  $F$  versus QPC bias energy  $eV_{\text{QPC}}$  for tunneling rate  $\Gamma_R$  equal to: (a) 0.03 meV, (b) 0.09 meV, (c) 0.15 meV. Parameters are the same as those in Fig. 2.

sibility and emphasize that the behaviors of the shot noise observed above are due to the QPC-induced backaction. When the QPC is decoupled to the DQD, the rates of electron tunneling to the right electrode from the two eigen-state channels  $|g\rangle$  and  $|e\rangle$  in the DQD are given from the dynamical equation (see Appendix) as  $\Gamma_R^{(g)} = \beta^2 \Gamma_R$  and  $\Gamma_R^{(e)} = \alpha^2 \Gamma_R$  with  $\alpha \neq \beta$ , in general, which will lead to *super*-Poissonian noise due to dynamically blocked channels.<sup>32–34</sup> However, as emphasized above, the assumption of a zero energy difference between two orbital levels (i.e.,  $\varepsilon = 0$ ) leads to  $\alpha = \beta$ . This leads to *sub*-Poissonian noise. This is exactly what we have seen in Fig. 3 when the backaction is small or absent.

However, if the bias voltage applied across the QPC increases, transitions between the two eigenstate channels are enhanced [see Eq. (12)] since  $\gamma_{\text{ex}} = \lambda |eV_{\text{QPC}} - \Delta|$  and  $\gamma_{\text{re}} = \lambda |eV_{\text{QPC}} + \Delta|$  when  $|eV_{\text{QPC}}| > \Delta$ . Then an electron can always switch to the faster tunneling channel. For example, if  $|g\rangle$  has a significantly higher tunneling rate, the transport will mainly proceed through  $|0\rangle \rightarrow |e\rangle \rightarrow |g\rangle \rightarrow |0\rangle$  and  $|0\rangle \rightarrow |g\rangle \rightarrow |0\rangle$ . That means that two eigen-state channels are not equivalent due to different effective tunneling rates. Thus, super-Poissonian noise as shown in Fig. 3 is an expected result. Interestingly, this backaction-induced changes of electron transport between bunching and anti-bunching behaviors should be realizable in the experiments with current technologies.

Note that the QPC-induced dephasing is not discussed above. This is because such dephasing with a rate  $\gamma_{\text{de}} (= \lambda |eV_{\text{QPC}}|)$  does not induce any transitions between different states of the DQD. Finally, we emphasize that, besides using an approach different from the previous ones (see Appendix), our results are derived from the FCS. The formalism can be extended to the study of high order correlations. We can also go beyond zero-frequency counting statistics presented above and further study frequency-dependent current statistics in the presence of charge-detector-induced backaction. This will fully characterize the fundamental tunneling processes including transient behaviors and will be discussed elsewhere.

#### IV. SUMMARY

We have studied the unavoidable charge-detector-induced backaction on the FCS of a biased DQD. We find that this backaction has profound effects on the FCS, namely enhancing the current through the DQD and changing the shot noise from a sub-Poissonian regime to a super-Poissonian one. These backaction effects can be experimentally examined by using the current technologies. Also, our results contribute to possible fine manipulation of quantum transport processes. When the measurement device is absent, we recover known results for both current and shot noise.

#### Acknowledgements

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#### Appendix A: Quantum dynamics of the DQD

Here, we derive a master equation to describe the quantum dynamics of the DQD, which is used to calculate the



FCS. In the eigenstate basis, the DQD Hamiltonian can be written as

$$H_{\text{DQD}} = \frac{\Delta}{2}(|e\rangle\langle e| - |g\rangle\langle g|), \quad (\text{A1})$$

where  $\Delta = \sqrt{\varepsilon^2 + 4\Omega^2}$  is the energy splitting of the two eigenstates of the DQD given by  $|g\rangle = \alpha|1\rangle - \beta|2\rangle$ , and  $|e\rangle = \beta|1\rangle + \alpha|2\rangle$ . In the interaction picture with the unperturbed Hamiltonian  $H_0 \equiv H_{\text{DQD}} + H_{\text{leads}} + H_{\text{QPC}}$ , the interaction Hamiltonian  $H_1 \equiv H_{\text{T}} + H_{\text{det}}$  can be written as

$$H_{\text{det}} = X(t)Y(t), \quad (\text{A2})$$

$$H_{\text{T}}(t) = \sum_s [c_{ls}^\dagger(\alpha a_g e^{-i\Delta t/2} + \beta a_e e^{i\Delta t/2})e^{i\omega_{ls}t} + \Upsilon_r^\dagger c_{rs}^\dagger \times (\alpha a_e e^{i\Delta t/2} - \beta a_g e^{-i\Delta t/2})e^{i\omega_{rs}t} + \text{H.c.}], \quad (\text{A3})$$

where

$$X(t) = \sum_{n=1}^3 U_n e^{i\omega_n t}, \quad (\text{A4})$$

$$Y(t) = \sum_{kq} V_{kq}^\dagger(t) + V_{kq}(t), \quad (\text{A5})$$

and  $U_1 = \zeta|e\rangle\langle g|$ ,  $U_2 = \zeta|g\rangle\langle e|$ ,  $U_3 = T - \zeta \cos \theta_{\varrho_z}$ ,  $\omega_1 = -\omega_2 = \Delta$ ,  $\omega_3 = 0$ ,  $V_{kq}(t) = c_{Dq}^\dagger c_{Sk} e^{-i(\omega_{Sk} - \omega_{Dk})t}$ .

Applying the Born-Markov approximation and tracing over the degrees of freedom of the QPC, the quantum dynamics of the DQD system in the Schrödinger picture is governed by the master equation,

$$\dot{\rho}(t) = -i[H_{\text{DQD}}, \rho(t)] + \mathcal{L}_d \rho(t) + \mathcal{L}_T \rho(t), \quad (\text{A6})$$

with

$$\mathcal{L}_d \rho(t) = \sum_{i,j=1, i \neq j}^3 \{ \mathcal{D}[P_i] \rho(t) + \mathcal{D}[P_i, P_j] \rho(t) \} \times \lambda [\Theta(eV_{\text{QPC}} - \omega_i) + \Theta(-eV_{\text{QPC}} - \omega_i)],$$

$$\begin{aligned} \mathcal{L}_T \rho(t) = & \alpha^2 \Gamma_L \mathcal{D}[a_g^\dagger] \rho(t) + \beta^2 \Gamma_L \mathcal{D}[a_e^\dagger] \rho(t) \\ & + \alpha^2 \Gamma_R \mathcal{D}[a_e \Upsilon^\dagger] \rho(t) + \beta^2 \Gamma_R \mathcal{D}[a_g \Upsilon^\dagger] \rho(t) \\ & + \alpha \beta \Gamma_L \{ [a_e^\dagger, \rho(t) a_g] + [a_g^\dagger \rho(t), a_e] \\ & + [a_g^\dagger, \rho(t) a_e] + [a_e^\dagger \rho(t), a_g] \} \\ & - \alpha \beta \Gamma_R \{ [a_e \Upsilon^\dagger, \rho(t) a_g^\dagger \Upsilon] + [a_g \Upsilon^\dagger \rho(t), a_e^\dagger \Upsilon] \\ & + [a_g^\dagger \Upsilon^\dagger, \rho(t) a_e^\dagger \Upsilon] + [a_e \Upsilon^\dagger \rho(t), a_g^\dagger \Upsilon] \}. \end{aligned}$$

Here,  $\rho(t)$  is the reduced density matrix of the DQD system. The superoperator  $\mathcal{D}$ , acting on any single or double operator, is defined as

$$\mathcal{D}[A] \rho \equiv A \rho A^\dagger - \frac{1}{2} A^\dagger A \rho - \frac{1}{2} \rho A^\dagger A, \quad (\text{A7})$$

$$\mathcal{D}[A, B] \rho \equiv \frac{1}{2} (A \rho B^\dagger + B \rho A^\dagger - B^\dagger A \rho - \rho A^\dagger B). \quad (\text{A8})$$

From Eq. (A6) and the relations

$$\langle n | \Upsilon_r^\dagger \rho \Upsilon_r | n \rangle = \rho^{(n-1)}, \quad \langle n | \Upsilon_r \rho \Upsilon_r^\dagger | n \rangle = \rho^{(n+1)}, \quad (\text{A9})$$

$$\langle n | \Upsilon_r^\dagger \Upsilon_r \rho | n \rangle = \rho^{(n)}, \quad \langle n | \Upsilon_r \Upsilon_r^\dagger \rho | n \rangle = \rho^{(n)}, \quad (\text{A10})$$

where  $n$  is the number of electrons that have tunneled to the right electrode, we obtain the  $n$ -resolved equation of motion for each reduced density matrix element:

$$\begin{aligned} \dot{\rho}_{00}^{(n)} = & -\Gamma_L \rho_{00}^{(n)} + \beta^2 \Gamma_R \rho_{gg}^{(n-1)} + \alpha^2 \Gamma_R \rho_{ee}^{(n-1)} \\ & - \alpha \beta \Gamma_R (\rho_{eg}^{(n-1)} + \rho_{ge}^{(n-1)}), \end{aligned} \quad (\text{A11})$$

$$\begin{aligned} \dot{\rho}_{gg}^{(n)} = & \alpha^2 \Gamma_L \rho_{00}^{(n)} - (\beta^2 \Gamma_R + \gamma_{ex}) \rho_{gg}^{(n)} + \gamma_{re} \rho_{ee}^{(n)} \\ & + (\frac{1}{2} \alpha \beta \Gamma_R + \eta \gamma_{de}) (\rho_{eg}^{(n)} + \rho_{ge}^{(n)}), \end{aligned} \quad (\text{A12})$$

$$\begin{aligned} \dot{\rho}_{ee}^{(n)} = & \beta^2 \Gamma_L \rho_{00}^{(n)} + \gamma_{ex} \rho_{gg}^{(n)} - (\alpha^2 \Gamma_R + \gamma_{re}) \rho_{ee}^{(n)} \\ & + (\frac{1}{2} \alpha \beta \Gamma_R - \eta \gamma_{de}) (\rho_{eg}^{(n)} + \rho_{ge}^{(n)}), \end{aligned} \quad (\text{A13})$$

$$\begin{aligned} \dot{\rho}_{eg}^{(n)} = & -i \Delta \rho_{eg}^{(n)} + \alpha \beta \Gamma_L \rho_{00}^{(n)} + (\frac{1}{2} \alpha \beta \Gamma_R + \eta \gamma_{ex}) \rho_{gg}^{(n)} \\ & + (\frac{1}{2} \alpha \beta \Gamma_R - \eta \gamma_{re}) \rho_{ee}^{(n)} - (\frac{1}{2} \Gamma_R + 2 \eta^2 \gamma_{de}) \rho_{eg}^{(n)} \\ & - \frac{1}{2} (\gamma_{ex} + \gamma_{re}) (\rho_{eg}^{(n)} - \rho_{ge}^{(n)}). \end{aligned} \quad (\text{A14})$$

In particular, we obtain the tunneling rates for two eigenstate channels  $\Gamma_L^{(g)} = \alpha^2 \Gamma_L$ ,  $\Gamma_L^{(e)} = \beta^2 \Gamma_L$ ,  $\Gamma_R^{(g)} = \beta^2 \Gamma_R$ , and  $\Gamma_R^{(e)} = \alpha^2 \Gamma_R$ .

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